difference between the responses of the two probes proving that the vortices are shed alternately.

The reduced frequency of shedding for all the cases vs the angle of attack is plotted in Fig. 4. In this figure we display data obtained from both facilities. There is a very clear dependence of the reduced frequency on the angle of attack. On the other hand, the influence of the Reynolds number is negligible.

The evidence presented here indicates that vortices are shed over delta wings at high angles of attack, just like the cases of other flat surfaces or bluff bodies. Once this aerodynamic phenomenon is set in motion, an aircraft will respond, and interaction between the aerodynamics and the wing attitude will lead to wing rock. However, it should be emphasized that this type of wing rock has not been studied so far. The basic difference with the well-known case is that for very large angles of attack, the flow is fully separated, even if the attitude of the aircraft is fixed.

Since the submission of this research Note, the present team has continued work on this project. Most recently, it was found that at intermediate angles of attack, simultaneous vortex shedding is also possible. The reader will find more information in a recent conference paper.[5]

References

[1]Rediniotis, O., Stapountzis, H., and Telionis, D. P., "Vortex Shedding over Bodies with Non-parallel Edges," Virginia Polytechnic Inst. and State Univ., Blacksburg, VA, Rept. VPI-E-88-89, Dec. 1988.

[2]Malcom, G. N., and Schiff, L. B., "Recent Developments in Rotary-Balance Testing of Fighter Aircraft Configurations at NASA Ames Research Center,"

[3]Brandon, J. M., and Shah, G. H., "Effect of Large Amplitude Pitching Motion on the Unsteady Aerodynamic Characteristics of Flat-plate Wings," AIAA Paper 88-4331-CP, 1988.

[4]LeMay, S. P., Batill, S. M., and Nelson, R. C., "Leading Edge Vortex Dynamics on a Pitching Delta Wing," AIAA Paper 88-2559-CP, 1988.

[5]Ayoub, A., and McLachlan, B. G., "Slender Delta Wing at High Angles of Attack—a Flow Visualization Study," AIAA Paper 87-1230, 1987.

[6]Rediniotis, O. K., Telionis, D. P., and Stapountzis, H., "Periodic Vortex Shedding Over Delta Wings," AIAA Paper 89-1923, 1989.

Postbuckling Analysis of Trusses with Various Lagrangian Formulations

Yeong-Bin Yang* and Liang-Jenq Leu†
National Taiwan University,
Taipei, Taiwan, Republic of China

Introduction

THIS Note is concerned with the incremental nonlinear analysis of elastic trusses involving large strains. First, it will be pointed out that use of the same material constants in an incremental anlaysis does not imply identical material behaviors for the total (TL), updated (UL), and general (GL) Lagrangian formulations. Second, in calculating the bar forces with an incremental analysis, the "total form," rather than the "incremental form," should be used, so as to avoid possible errors.

Received May 11, 1989; revision received June 30, 1989.

*Professor, Department of Civil Engineering

†Graduate Research Assistant, Department of Civil Engineering.

In an incremental formulation, three configurations are needed to describe the motion of a body, i.e., the initial undeformed configuration C_0, current (known) deformed configuration C_1, and a neighboring (desired) deformed configuration C_2. In the TL formulation, C_0 is chosen as the reference, whereas in the UL formulation C_1 is used. Both the TL and UL formulations can be considered as special cases of a GL formulation that adopts an arbitrary known configuration C_m between C_0 and C_1 as the reference.[1] General discussions of the Lagrangian procedures can be found in Refs. 2 and 3. In this Note, left superscripts and subscripts will be used to indicate the occurring and measuring configurations of a quantity, respectively. An incremental quantity between C_1 and C_2 will be denoted with no left superscripts. The following symbols are used in the present discussions: A = sectional area of bar; L = bar length; S = second Piola-Kirchhoff stress; x = axial coordinate, ϵ = Green-Lagrange strain; ρ = specific mass of materials; and τ = Cauchy stress.

Incremental Constitutive Laws

Conventionally, the material law has been expressed in the following incremental form:

$$\text{TL}: {}_0S = F'\left({}^1_0\epsilon\right){}_0\epsilon \tag{1a}$$

$$\text{UL}: {}_1S = f'\left({}^1_0\epsilon\right){}_1\epsilon \tag{1b}$$

$$\text{GL}: {}_mS = f'\left({}^1_0\epsilon\right){}_m\epsilon \tag{1c}$$

where f is a single-valued function and (′) is the derivative with respect to the strain ${}^1_0\epsilon$. It is possible to integrate the preceding incremental laws to obtain expressions in terms of the total stresses and strains. By so doing, one may see that the material behaviors implied by Eqs. (1) are very different for large strain problems, as will be demonstrated later.

Formulations with Infinitesimal Step Sizes

Consider a bar that is stretched with an infinitely large number of infinitesimal steps from C_0 to C_2. Let C_i denote an arbitrary configuration of the bar between C_0 and C_2.

TL Formulation

In this case, the strain and stress increments, ${}_0\epsilon$ and ${}_0S$, should be interpreted as the increments from C_i to an infinitessimally close neighboring state. The total stress can be obtained by integration:

$$ {}^2_0S = \int {}_0S = \int_0^{{}^2_0\epsilon} f'\left({}^i_0\epsilon\right){}_0\epsilon \tag{2}$$

Noting that ${}^2\rho/{}^0\rho = {}^0A\,{}^0L/({}^2A\,{}^2L)$ for conserved mass and $\partial^2x/\partial^0x = {}^2L/{}^0L$, one can relate the Cauchy stress ${}^2\tau$ to the second Piola-Kirchhoff stress ${}^2_0S$ as follows[2,3]:

$$2\tau = {}^2_0S\,{}^0A\,{}^0L/({}^2A\,{}^0L) \tag{3}$$

Consequently, the axial force ${}^2_2F$ at C_2 can be obtained:

$${}^2_2F = 2_\tau{}^2A = {}^2_0S\,{}^0A\,({}^2L/{}^0L) \tag{4}$$

UL Formulation

For the purpose of obtaining closed-form expressions, one may convert the UL formulations, with moving coordinates, into an equivalent TL formulation, with a fixed reference. The stress increment for the UL formulation is ${}_iS = E_i\epsilon$, which is equivalent to ${}_0S = {}_0E^U_{eq0}\epsilon$ in the TL formulation, based on the transformation rule for constitutive tensors[2,3]:

$$ {}_0E^U_{eq} = f'\left({}^i_0\epsilon\right)({}^iA/{}^0A)({}^0L/{}^iL)^3 \tag{5} $$

Accordingly, the equivalent stress for the TL formulation is

$$ {}^2_0S = \int_0^{{}^2_0\epsilon} {}_0E^U_{eq0}\epsilon \tag{6} $$

and the member force ${}^2_2F$ can be calculated from Eq. (4) as well.

GL Formulation

Again, the equivalent TL formulation will be used, in which the equivalent tangent modulus is

$$ {}_0E^G_{eq} = f'\left({}^i_0\epsilon\right)({}^mA/{}^0A)({}^0L/{}^mL)^3 \tag{7} $$

By calculating the equivalent stress as

$$ {}^2_0S = \int_0^{{}^2_0\epsilon} {}_0E^G_{eq0}\epsilon \tag{8} $$

the member force at C_2 can be obtained from Eq. (4).

Formulations with Finite Step Sizes

In the following, the increment step from C_1 to C_2 will be assumed to be of finite size. For the GL formulation, a reference configuration C_m between C_0 and C_1 will be selected. Realizing that $\partial^2x_1/\partial^mx_1 = {}^2L/{}^mL$, one writes the Cauchy stress in terms of the second Piola-Kirchhoff stress as

$$ {}^2\tau = \frac{{}^2\rho}{{}^m\rho}\left(\frac{{}^2L}{{}^mL}\right)^2\left({}^1_mS + {}_mS\right) \tag{9} $$

With the strain increment ${}_m\epsilon$ defined as

$$ {}_m\epsilon = ({}^2L^2 - {}^1L^2)/(2\,{}^mL^2) \tag{10} $$

the stress increment ${}_mS$ can be obtained from Eq. (1c). Assuming ${}^2\rho/{}^m\rho = {}^mA\,{}^mL/({}^2A\,{}^2L)$ for conserved mass, the member force becomes

$$ {}^2_2F = {}^2\tau\,{}^2A = \left({}^1_mF + {}_mF\right)\,{}^2L/{}^mL \tag{11} $$

where ${}^1_mF$ and ${}_mF$ are defined as follows:

$$ {}^1_mF = {}^1_1F\,{}^mL/{}^1L \tag{12} $$

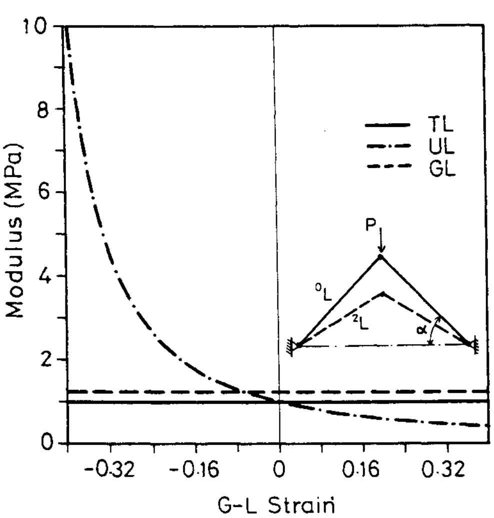

Fig. 1 Modulus-strain curve for case A materials.

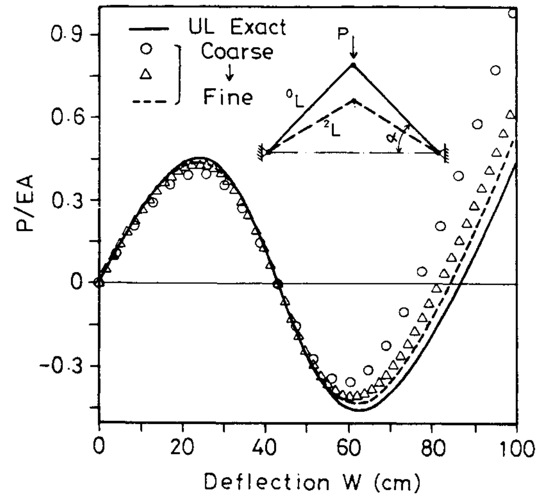

Fig. 2 UL solution for case A materials.

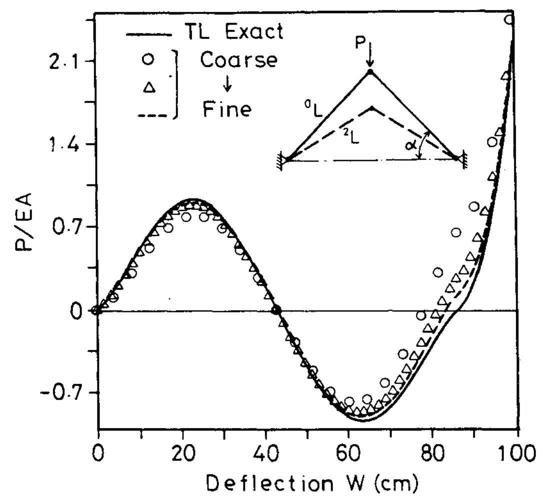

Fig. 3 TL solution for case B materials.

$$ {}_mF = f'\left({}^1_0\epsilon\right){}^mA\,({}^2L^2 - {}^1L^2)/(2\,{}^mL^2) \tag{13} $$

Equation (12) can be proved by letting C_2 coincide with C_1 in Eq. (11). In this case, ${}^2L = {}^1L$, ${}_mF = 0$, and ${}^2_2F = {}^1_1F = {}^1_mF\,{}^1L/{}^mL$; so the equation is proved.

By letting C_m equal C_0, the following equations can be obtained from Eq. (11) for the TL formulation:

$$ {}^2_2F = \left({}^1_0F + {}_0F\right){}^2L/{}^0L \tag{14} $$

where the initial bar force ${}^1_0F$ and bar force increment ${}_0F$ are

$$ {}^1_0F = {}^1_1F\,{}^0L/{}^1L \tag{15} $$

$$ {}_0F = f'\left({}^1_0\epsilon\right){}^0A\,({}^2L^2 - {}^1L^2)/(2\,{}^0L^2) \tag{16} $$

Also by letting C_m equal C_1, the following equations can be obtained for the UL formulation:

$$ {}^2_2F = \left({}^1_1F + {}_1F\right){}^2L/{}^1L \tag{17} $$

where the bar force increment ${}_1F$ is

$$ {}^2_1F = f'\left({}^1_0\epsilon\right){}^1A\,({}^2L^2 - {}^1L^2)/(2\,{}^1L^2) \tag{18} $$

The equations derived in this section for the bar forces [Eqs. (11), (14), (17),] will be referred to as the "incremental form," which are piecewise linear, whereas those derived previously based on the integration process [Eqs. (4), (2), (6), (8)] will be termed the "total form," which are fully nonlinear.

Numerical Examples

The two-member truss shown in Fig. 1 will be studied, assuming ${}^0A = {}^1A = {}^2A = 10$ cm^2, ${}^0L = 50$ cm, and ${}^mL = 25(1 + \cos\alpha)$ cm. Two types of materials will be considered: 1) Case A—linearly elastic materials (in TL sense):

$$ f'\left({}^i_0\epsilon\right) = E \tag{19} $$

and 2) Case B – nonlinearly elastic materials:

$$ f'\left({}^i_0\epsilon\right) = E + 2E_a\left|{}^i_0\epsilon\right| \tag{20} $$

where E and E_a are the material constants. For the truss with $\alpha = 30$ deg and $E = 1$ MPa, the tangent moduli implied by the TL, UL, and GL formulations, i.e., Eqs. (19), (5), and (7), are drawn in Fig. 1. As can be seen, the material implied by the UL formulation is in fact nonlinear. It tends to soften in tension and to harden in compression. Although the materials implied by the TL and GL formulations are linear, they are not identically the same.

In the incremental load-deflection analysis, the truss is assumed to have an angle of $\alpha = 60$ deg with $E_a = 10E = 10$ MPa. The method of solution adopted herein is the displacement control method. Both the "total form" and "incremental form" will be used for calculating the member forces. The results obtained with the "total form" will be referred to as exact, as they are not affected by the step sizes used. Figures 2 and 3 show the solutions obtained with the UL formulation for case A materials and those by the TL formulation for case B materials, respectively. Additional examples including more complicated trusses may be found in Ref. 4.

With regard to the previous solutions, the following observations can be made: 1) For formulation with "nonlinear" equivalent tangent modulus in the TL sense, the load-deflection curves obtained with the "incremental form" are step-size dependent (Figs. 2 and 3), because of the linearization of the material laws in each step; 2) For formulations with "linear" equivalent tangent modulus in the TL sense, the solutions obtained with the "incremental form" coincide with the exact ones, which are step-size independent[4]; and 3) All of the solutions that are step-size dependent converge to the exact ones as the step sizes are reduced.

Conclusions

In postbuckling analysis of trusses with large strains, the use of identical constants in the material laws [Eqs. (1)] does not imply identical tangent modulus for the TL, UL, and GL formulations. Two procedures have been presented for calculating the bar forces. One is the "total form," which is fully nonlinear, and the other is the "incremental form," which is a piecewise linear approximation of the former. Using the "total form" in a nonlinear analysis will result in solutions that are exact and step-size independent. In contrast, using the "incremental form" for bar forces will generate solutions that are step-size dependent. For problems with large strains, the errors involved with the latter procedure can be significant.

Acknowledgment

This research was sponsored by the National Science Council of the Republic of China under Grant NSC76-0414-P002-018.

References

[1]Cescotto, S., Frey, F., and Fonder, G., "Total and Updated Lagrangian Descriptions in Nonlinear Structural Analysis: A Unified Approach," *Energy Methods in Finite Element Analysis,* edited by R. Glowinski, E. Y. Rodin, and O. C. Zienkiewicz, Wiley, New York, 1979, Chap. 15, p. 283–296.

[2]Bathe, K.-J., *Finite Element Procedures in Engineering Analysis,* Prentice-Hall, Englewood Cliffs, NJ, 1982, Chap. 6.

[3]Washizu, K., *Variational Methods in Elasticity and Plasticity,* 3rd ed., Pergamon, Oxford, UK, 1982.

[4]Leu, L. J., "Nonlinear Theory of Trusses and Frames," M.S. Thesis, Dept. of Civil Engineering, National Taiwan Univ., Taipei, Taiwan, China, 1989 (in Chinese).

Two-Level Approximation Method for Stress Constraints in Structural Optimization

Garret N. Vanderplaats*
and
Srinivas Kodiyalam†
VMA Engineering
Goleta, California

Introduction

RECENT trends in stress-constrained optimization have established that approximation of the element nodal forces better retains the essential nonlinearities of the true (original) constraints than the usual approach of linearizing the actual constraints with respect to the design variables.[1,2] However, for continuum structures, this increases the cost of the approximate optimization phase because numerous element level solutions of the equilibrium equations are required for stress recovery.

Here both the force approximation and the direct-stress approximation methods are considered together to create a two-level scheme for stress constraints. This is an extension of the work reported in Ref. 3.

Mathematical Problem Statement

The mathematical programming problem is stated as follows.

Minimize

$$ F(X) \qquad \text{weight of the structure} \tag{1} $$

Subject to

$$ g_j\,(X) \le 0 \qquad j = 1, M \qquad \text{stress constraints} \tag{2} $$

$$ \mathbf{X}_i^L \le \mathbf{X}_i \le \mathbf{X}_i^U \qquad \mathbf{i} = 1, N \qquad \text{side constraints} \tag{3} $$

Received May 15, 1989; revision received August 25, 1989.

*President. Associate Fellow AIAA.

†Research Engineer. Member AIAA.